

Research Article

Study of Channel Capacity Optimization of Discrete Channel Based on Normalized Tangent Function

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Channel capacity is the rate limit of information transmission; many researchers strive to find the maximum channel transmission rate while balancing the performance of the whole communication system and making it close to the channel capacity. We know in the discrete channel that, when the BER of the system is small enough, the transmission rate of the channel will be extremely close to the channel capacity. According to Shannon theory, the common way to increase the channel capacity is to increase the system bandwidth and the input signal power. However, the bandwidth resources are extremely limited, so it is impractical to expand the system bandwidth, and at the same time, increasing the input signal power is impractical as well, because it will reduce the economy of the communication system. In this paper, we proposed a normalized tangent function (NorTF) which can optimize the discrete channel probability distribution, reduce the BER of the channel, and improve the channel capacity of the discrete channel, which need not expand bandwidth and increase the signal input power of the system. Through theoretical analysis and experimental verification of this method, we found that the channel capacity of the improved system was indeed much higher than that of the preimproved system, so the method proposed in this paper has some theoretical guiding significance and is feasible.

1. Introduction

The fifth-generation (5G) mobile communication has appeared in our lives; compared to the fourth-generation mobile communication (4G), it has 1000 times system capacity, 100 times data rate, 3-5 times spectrum efficiency, and 10-100 times energy efficiency [1]. Therefore, 5G will provide all-time services for people's living, work, leisure, transportation, etc. So 5G must have higher channel capacity; thus, how to improve the channel capacity is an important research topic in 5G and even the next generation of the mobile communication system.

We know that the MIMO system is the earlier channel capacity optimization method; it need not extra bandwidth and extra send power, so the MIMO technology has become a research hotspot in the field of wireless communication. Lots of researchers have studied MIMO systems and their

channel capacity, and they all believed that the MIMO system will increase the signal-to-noise ratio (SNR) gain and will certainly increase the channel capacity of the communication system. But some researchers [2] found that the IRFC constructed by MIMO theory on the baseband does not fit the physical concept of SDM, so the SDM defined by MIMO cannot be implemented physically. At the same time, the study also found that MIMO uses multiple antennas to form multipath transmission channels, which not only wastes power resources but also makes the signal reception environment more harsh and the required signal processing process more complex.

Beside the MIMO theory, some researchers [3] proposed the multiuser molecular communication model and the method which can make the channel capacity maximized by jointly optimizing the power-constrained precoder at the base station and the unit modulus-constrained phase

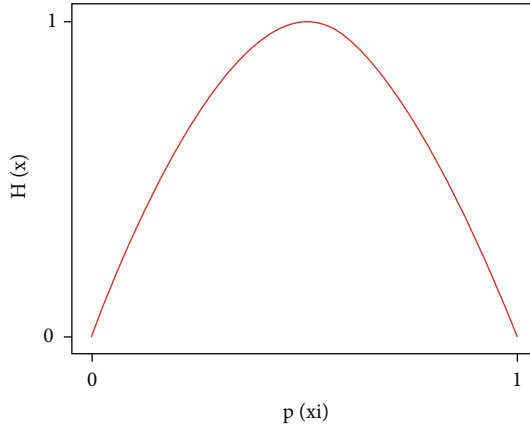
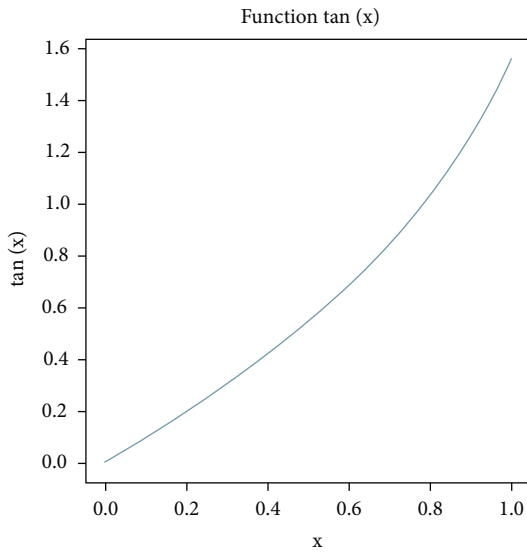


FIGURE 1: Entropy function.

FIGURE 2: Function $\tan(x)$.

shifter at the IRS in the IRS-assisted multiuser downlink communication. Intelligent reflecting surface (IRS) can offer unprecedented channel capacity gains.

Inspired by these methods, in this paper, we proposed a normalized tangent function (NorTF) to reduce the BER of discrete channel to optimize the channel capacity of the communication system.

The NorTF method proposed in this paper can be used after any optimized discrete channel, which can make the discrete channel get double optimization. At the same time, this method can be easily achieved by software and has nearly zero communication cost.

2. Common Methods to Improve Channel Capacity

In 1998, Foschini and Gans proposed the MIMO system structure and theory [4] and mathematically proved that the system channel capacity would be increased without needing to increase the channel bandwidth. Since then, MIMO technology has become a research hotspot in the

field of wireless communication. Currently, lots of researchers have studied the MIMO systems and their channel capacity [5–8]. They all believed that the MIMO system will increase the signal-to-noise ratio (SNR) gain and will certainly increase the channel capacity of the communication system because it uses PCA transmission and multiple receive antennas. Haiyang et al. [2], found that the IRFC constructed by MIMO theory [2] on the baseband does not fit the physical concept of SDM, so the SDM defined by MIMO cannot be implemented physically. The physical model derived from the MIMO mathematical model will form a large area of electromagnetic wave blind area. At the same time, the study also found that MIMO uses multiple antennas to form multipath transmission channels, which not only wastes power resources but also makes the signal reception environment harsher and the required signal processing process more complex. They constructed the SHPCA system based on the Shannon application principle and phased antenna array theory, whose channel capacity is directly ratioed to the antenna number and SDM times and is more efficient than Shannon theory.

Besides MIMO theory, there are also other methods to improve channel capacity.

Aim to switch-controlled modulation for diffusion, Cheng et al. [3] proposed a multiuser molecular communication model. In the diffusive multi-user molecular communication model, the released molecules follow Brownian motion rules, the Inter-Symbol Interference (ISI) and Inter-User Interference (IUI) among the molecules inevitably exist. In order to increase the channel capacity of system, they research on the molecular communication model. For the diffusive multi-user molecular communication model with On-Off Keying (OOK) modulation, the ISI and IUI were analyzed. The minimum average error probability criterion was used to obtain the optimal decision threshold in the detection process for the receiver nanomachine, thus the channel capacity of the molecular communication model based on IUI can be improved.

In the paper channel capacity optimization in the IRS-aided multiuser communication system, Wang et al. [9] proposed the method which can make the channel capacity maximized by jointly optimizing the power-constrained precoder at the base station and the unit modulus-constrained phase shifter at the IRS in the IRS-assisted multiuser downlink communication. Intelligent reflecting surface (IRS) can offer unprecedented channel capacity gains, which consist of passive reflecting elements that induce phase shifts on the imping electromagnetic waves to smartly reconfigure the signal propagation environment.

3. Find Out the Question

We find it complicated that whatever method is used to improve the transmission capacity of the communication system, the channel model itself is not fundamentally optimized, so the transmission capacity of the single channel itself is not improved. So, according to the characteristics of the discrete channel model, this paper proposed the normalized tangent function based on the softmax method to

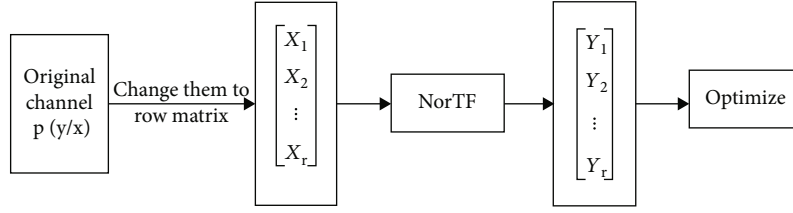


FIGURE 3: Optimization of discrete symmetric channel.

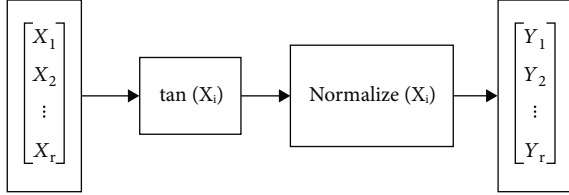


FIGURE 4: NorTF method.

improve the channel model and improve the channel capacity.

We find that this method can fundamentally optimize the single discrete channel model and significantly improve the channel capacity.

In this paper, we have mainly done the following work.

First, we design the optimization system model; second, we analyze the optimization process; and last, we calculate the optimization result and compare the channel capacity before and after the optimization.

4. Optimization Model

4.1. Channel Capacity of Discrete Symmetric Channel. With the development of digital communication, the channel research is mainly based on discrete channel. So we take the discrete symmetric channel as the main research object in this paper.

We know that channel capacity is the largest transmission rate of a channel, and we can write it as $C = \max_{p(x)} I(X; Y)$,

where C is the channel capacity, $I(X; Y)$ is the information content, $p(x)$ is the probability of input x , X is the input, and Y is the output [10].

The channel capacity of a discrete symmetric channel with r input symbols and s output symbols will be written as

$$C = \max_{p(x)} I(X; Y) = \log s - H(p'_1, p'_2, \dots, p'_s). \quad (1)$$

In formula (1), p'_1, p'_2, \dots, p'_s are line elements of any one line in channel matrix.

In the analysis of formula (1), we find the channel capacity of discrete symmetric channel is only related to the entropy $H(p'_1, p'_2, \dots, p'_s)$ when the number of output elements is determined. Obviously, the larger the entropy $H(p'_1, p'_2, \dots, p'_s)$, the smaller the channel capacity C . Therefore, in order to increase the channel capacity C , we need to opti-

mize the line elements distribution p'_1, p'_2, \dots, p'_s and make the entropy $H(p'_1, p'_2, \dots, p'_s)$ become smaller.

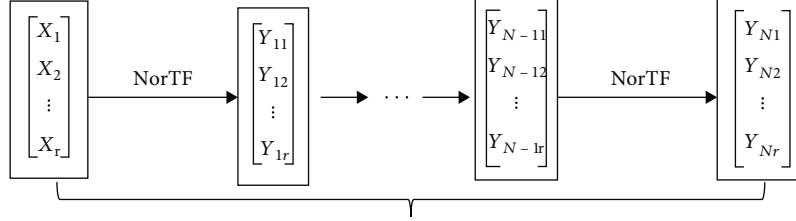
We know that the entropy in communication system comes from the entropy function in thermodynamics. The entropy function reflects the uniformity of its state. The curve of entropy function with the distribution of the state is shown in Figure 1 [11].

In Figure 1, we can see that the more uniform the distribution of each element, the greater its entropy. Therefore, in order to optimize the channel model and increase the channel capacity of the communication channel, it is necessary to adjust the distribution of row elements in the channel transfer matrix p'_1, p'_2, \dots, p'_s to increase the probability gap and reduce the entropy $H(p'_1, p'_2, \dots, p'_s)$. So, the real work of optimizing the model is to adjust the probability distribution of row elements.

4.2. Optimization Theory Analysis. According to the analysis in 4.1, we know that the BER in the channel transfer matrix is the only factor affecting the channel capacity when the number of input and output symbols is determined. Thus, the only way to improve the channel capacity is to find a way to reduce the BER in the channel transfer matrix. Therefore, we should make certain changes to the probability distribution in the channel transfer matrix without changing the overall channel model structure, that is, we should make the large value in the probability distribution larger and the small value smaller, and at the same time, we must ensure that the sum of the probabilities of each row is still 1. According to this analysis, we find the softmax function [12] often used in multi classification tasks in deep learning meets this requirement.

Inspired by the softmax function, we find that when $0 < x < 1$, $\tan(x)$ can meet the requirements of making the large value larger and the small value smaller. When $0 < x < 1$, $\tan(x)$ is shown in Figure 2. If we use the normalized tangent function (NorTF), it will fully meet the above requirements. Thus, this paper proposed the NorTF method to optimize the channel model of discrete symmetric channel, which will improve the channel capacity of discrete symmetric channel.

4.3. Optimization Model. In order to process easily, we transform the channel transfer matrix into a row matrix. First, we extracted the original channel transfer matrix by rows and expressed each row as an s -dimensional row vector, so the original channel transfer matrix can be represented as a row matrix, in which every row is an s -dimensional row

FIGURE 5: n times optimization for channel model.

$$\begin{bmatrix} \bar{p} & \frac{p}{s-1} & \cdots & \frac{p}{s-1} \\ \frac{p}{s-1} & \bar{p} & \cdots & \frac{p}{s-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p}{s-1} & \frac{p}{s-1} & \cdots & \bar{p} \end{bmatrix}$$

FIGURE 6: Channel model of strong symmetric channel.

vector. Then we used NorTF method to process the row matrix and obtain the optimized matrix. The whole optimization process is shown in Figure 3.

The NorTF method process is shown in Figure 4.

According to above optimization method, we can use normalized tangent function (NorTF) to optimize the origi-

nal channel model repeatedly until the channel capacity approaches our expected value. Its optimization structure is shown in Figure 5.

4.4. Analysis of Experimental Results. For convenience, we use strong symmetric channel for theoretical derivation in this part. The channel model of the strong symmetric channel is shown in Figure 6 [13].

In Figure 6, \bar{p} is the right transfer probability, p is the wrong transfer probability, and s is the output symbol number. According to the above NorTF optimization method, we optimize the model of Figure 5. Extract one row of the above channels and make it as an s -dimensional row vector X_i , $i, j = 1, 2, \dots, s$. Optimize every row vector X_i , and we can get a new row vector Y_i as

$$Y_i = \left[\frac{\tan(\bar{p})}{\tan(\bar{p}) + (s-1)\tan(p/(s-1))}, \frac{\tan(p/(s-1))}{\tan(\bar{p}) + (s-1)\tan(p/(s-1))}, \dots, \frac{\tan(p/(s-1))}{\tan(\bar{p}) + (s-1)\tan(p/(s-1))} \right]. \quad (2)$$

The channel model after 1-time optimization is shown in Figure 7.

According to the optimized channel model, we can calculate the channel capacity C , and C is shown as

$$C = \log s - H \left(\frac{\tan(\bar{p})}{\tan(\bar{p}) + (s-1)\tan(p/(s-1))}, \frac{\tan(p/(s-1))}{\tan(\bar{p}) + (s-1)\tan(p/(s-1))}, \dots, \frac{\tan(p/(s-1))}{\tan(\bar{p}) + (s-1)\tan(p/(s-1))} \right). \quad (3)$$

According to above analysis, we know that $(\tan(\bar{p})) / (\tan(\bar{p}) + (s-1)\tan(p/(s-1))) > \bar{p}$ and $((\tan(p/(s-1))) / (\tan(\bar{p}) + (s-1)\tan(p/(s-1)))) < p/(s-1)$, so the value of entropy function will become smaller, that is $H((\tan(\bar{p})) / (\tan(\bar{p}) + (s-1)\tan(p/(s-1))), (\tan(p/(s-1))) / (\tan(\bar{p}) + (s-1)\tan(p/(s-1))), \dots, (\tan(p/(s-1))) / (\tan(\bar{p}) + (s-1)\tan(p/(s-1)))) < H(\bar{p}, p/(s-1), \dots, p/(s-1))$; thus, the channel capacity will become larger, and the channel model will be optimized.

5. Experimental Verification

Compare and analyze the original channel and the optimized channel of the symmetric channel on the channel model and the channel capacity.

We choose the following four symmetric channel models to verify above optimization method. The comparison between the original channel and the optimized channel is shown in Figure 8.

Observing the channel model of nonoptimized channel and the channel model of optimized channel, we find that the large value in the channel transfer matrix of nonoptimized channel will become the larger value in the channel transfer matrix of optimized channel and the small value will become the smaller value, which shows that the method we proposed (NorTF) can optimize the discrete channel probability distribution and reduce the BER of the discrete channel.

From left to right, we named the above four channels as channel 1, channel 2, channel 3, and channel 4. The

$$\begin{bmatrix}
 \frac{\tan(\bar{p})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} & \frac{\tan(\frac{P}{s-1})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} & \dots & \frac{\tan(\frac{P}{s-1})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} \\
 \frac{\tan(\frac{P}{s-1})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} & \frac{\tan(\bar{p})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} & \dots & \frac{\tan(\frac{P}{s-1})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} \\
 \vdots & \vdots & \ddots & \vdots \\
 \frac{\tan(\frac{P}{s-1})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} & \frac{\tan(\frac{P}{s-1})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})} & \dots & \frac{\tan(\bar{p})}{\tan(\bar{p}) + (s-1)\tan(\frac{P}{s-1})}
 \end{bmatrix}$$

FIGURE 7: Discrete symmetric channel model after 1-time optimization.

$$\begin{bmatrix} 0.167 & 0.167 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.167 & 0.167 \end{bmatrix}
 \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}
 \begin{bmatrix} 0.75 & 0.125 & 0.125 \\ 0.125 & 0.75 & 0.125 \\ 0.125 & 0.125 & 0.75 \end{bmatrix}
 \begin{bmatrix} 0.938 & 0.031 & 0.031 \\ 0.031 & 0.938 & 0.031 \\ 0.031 & 0.031 & 0.938 \end{bmatrix}$$

(a) Original channel

$$\begin{bmatrix} 0.164 & 0.164 & 0.336 & 0.336 \\ 0.336 & 0.336 & 0.164 & 0.164 \end{bmatrix}
 \begin{bmatrix} 0.517 & 0.242 & 0.242 \\ 0.242 & 0.517 & 0.242 \\ 0.242 & 0.242 & 0.517 \end{bmatrix}
 \begin{bmatrix} 0.788 & 0.106 & 0.106 \\ 0.106 & 0.788 & 0.106 \\ 0.106 & 0.106 & 0.788 \end{bmatrix}
 \begin{bmatrix} 0.956 & 0.022 & 0.022 \\ 0.022 & 0.956 & 0.022 \\ 0.022 & 0.022 & 0.956 \end{bmatrix}$$

(b) Optimized channel

FIGURE 8: The comparison between the original channel and the optimized channel.

TABLE 1: The experimental values of the key steps in the optimization process of these four channels.

Items channel	Row element p_i	C of original channel	$\tan(p_i)$	Optimization result q_i	C of optimized channel	Improvement of channel capacity
Channel 1	0.167	0.081	0.169	0.164	0.087	7.407%
	0.167		0.169	0.164		
	0.333		0.346	0.336		
	0.333		0.346	0.336		
Channel 2	0.5	0.085	0.546	0.517	0.103	21.176%
	0.25		0.255	0.2415		
	0.25		0.255	0.2415		
	0.75		0.932	0.788		
Channel 3	0.125	0.524	0.126	0.106	0.626	19.466%
	0.125		0.126	0.106		
	0.938		1.364	0.956		
	0.031		0.031	0.022		
Channel 4	0.031	1.188	0.031	0.022	1.283	7.997%
	0.031		0.031	0.022		

experimental values of the key steps in the optimization process of these four channels are shown in Table 1.

6. Conclusion

We can see from Figure 8 and Table 1 that the channel capacity of optimized channel is much higher than that of nonoptimized channel, which shows that the NorTF method proposed in this paper is feasible. We can see from Figure 8

and Table 1, the channel capacity of optimized channel and the channel BER of nonoptimized channel has the following relationship, take 0.5 as the cut-off point of the channel BER of nonoptimized channel, when the channel BER is 0.5, the channel capacity of optimized channel will be increased significantly, when the channel BER become smaller from 0.5, this increase will be gradually reduced. It shows the fact that the larger BER of nonoptimized channel, the larger increase the channel capacity of optimized channel.

From above analysis we find that experimental result is perfectly agreed with the theoretical analysis.

For channel with large BER, we can perform multiple iterative optimizations to achieve our expected channel capacity.

From theoretical analysis to experimental verification, we find that optimizing the channel model based on the NorTF method actually fundamentally reduces the BER in channel, so we can infer that this method is applicable not only to the discrete symmetric channels, but also for all discrete channels.

But for continuous channel, this method is absolutely not applicable; we should do another research to optimize its channel capacity.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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