

## Research Article

# New Solutions of Nonlinear Equations Constrained by Bounded Variables under Edge Computation

Wangxun Yu  and Aiju Wang

College of Mathematics and Statistics, Xinyang University, Xinyang 464000, China

Correspondence should be addressed to Wangxun Yu; yuwangx@yeah.net

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Aiming at the problem of low global convergence and local convergence rate of trust region interior points of bounded variable constrained nonlinear equations, a trust region interior point algorithm for bounded variable constrained nonlinear equations under edge calculation is designed. By constructing the basic function form of nonlinear equations constrained by bounded variables, the boundary of nonlinear equations is determined by Gauss Newton iterative process to ensure the global convergence of changes; solve the original objective function, analyze the trust region subproblem of the unconstrained optimization problem, and generate an acceptable region. The region is generated through two-dimensional example interpretation. The interior points in the acceptable region are determined by the primal dual interior point method, and the interior points in the acceptable region are optimized. With the help of edge calculation, the trust region interior point programming model of bounded variable constrained nonlinear equations is designed to realize the algorithm design. The experimental results show that the designed algorithm can improve the trust region interior point global convergence and local convergence rate of nonlinear equations with bounded variable constraints.

## 1. Introduction

Nonlinear programming is an important research branch of optimization theory and method, and it is widely used in engineering practice, management decision-making, and many other fields. Nonlinear optimization mainly includes unconstrained optimization, convex optimization, Lagrange multiplier theory and algorithm, and duality theory and method and studies gradient method, Gauss Newton method, conjugate direction method, and then feasible direction method, gradient projection method, conditional gradient method, coordinate block descent method, and other algorithms for solving beam reduction problem [1, 2]. Among them, in the unconstrained problem, the basic idea of the gradient method is that the negative gradient direction of an iteration is the direction in which the objective function value drops the fastest at this point. Therefore, taking the negative gradient direction as the search direction is also called the steepest descent method, and its convergence speed is linear at most [3]. The idea of the conjugate gradient method is to use the gradient

of the objective function to gradually generate the conjugate direction as the line search direction, generally taking the negative gradient direction as the initial direction, and then construct a new search direction from the conjugation. The conjugate gradient method has quadratic termination for the quadratic function. In the solution of nonlinear system, the Newton method is an important and commonly used iterative method. Its basic idea is to linearize the nonlinear equations step by step. Under certain conditions, it has the property of quadratic convergence [4]. The basic idea of the trust region algorithm is to give a trust region in each iteration, solve a subproblem in this trust region, take the solution as a trial, and then decide whether to accept the trial step according to a certain standard. Generally, the trust region of the next iteration is determined based on the degree of simulation of the original function. Trust region algorithm has fast convergence speed [5].

In applied mathematics and engineering technology, we need to solve nonlinear equations in many cases. For example, the section characteristics of structural members, the



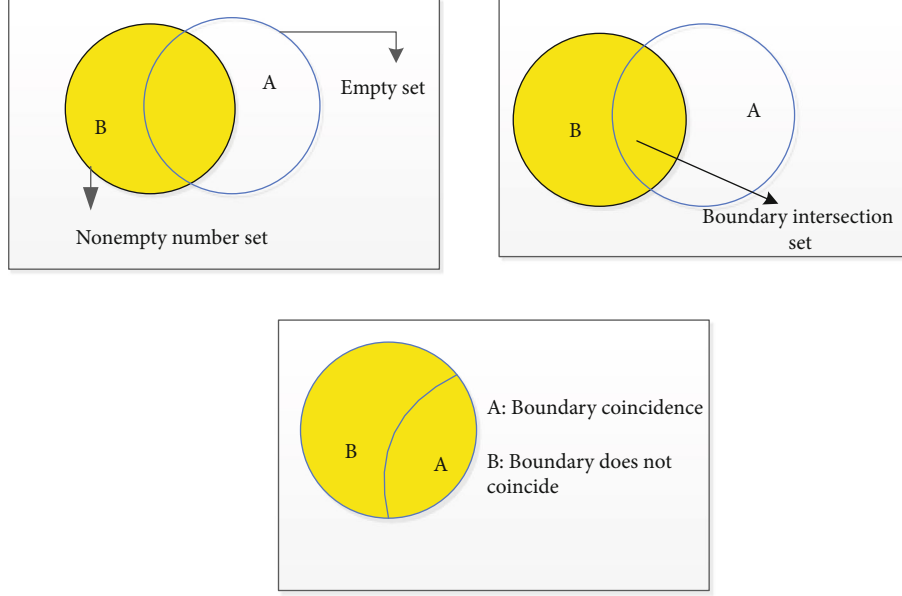


FIGURE 1: Schematic diagram of upper and lower bounds and set coincidence of nonlinear equations with bounded constraints.

In Figure 1, multivariate interpolation function technology and trust region strategy are combined to solve the problem of bounded constrained nonlinear equations. Firstly, the corresponding interpolation model is established for each function in the objective function. Such approximation does not require additional function calculation. In general, many interpolation points need to be used to determine a completely quadratic interpolation model [14]. Instead, the algorithm only needs  $2n + 1$  interpolation points when establishing the interpolation model, while the trust domain subproblem considers the second-order Taylor expansion of the polynomial interpolation model to introduce an affine transformation matrix, so that the problem has only one ellipsoid constraint. However, when the strict inner point is feasible, it is difficult to obtain the descent direction of the objective function by solving the trust domain subproblem [15].

In order to obtain a feasible descent step, the trust region subproblem with strict feasible constraints needs to be solved repeatedly. Therefore, the total calculation cost is very large. In order to overcome this disadvantage, the upper and lower bounds of the nonlinear equations with bounded constraints are determined by the Gauss Newton method. This method does not need to calculate the trust region subproblem repeatedly in each iteration, which reduces the amount of calculation and improves the purpose. The use of backtracking line search can ensure the feasibility of strict interior points.

Firstly, the boundary problem of nonlinear equations with bounded constraints is equivalent to the nonlinear least squares problem; that is,

$$\min_{x \in R^n} \vartheta(x) = \frac{1}{2} \|V(x)\|^2, \quad (6)$$

where the gradient of  $\vartheta(x)$  is

$$\Delta\vartheta(x) = p(x)^T V(x). \quad (7)$$

The Hesse matrix of  $\vartheta(x)$  is

$$G(x) = p(x)^T V(x) + s(x). \quad (8)$$

Among them,

$$s(x) = \sum_{i=1}^n f_i(x) \Delta^2 f_i(x). \quad (9)$$

At this time, the second order term  $s(x)$  in  $G(x)$  can be ignored, and then the quadratic model of the objective function is as follows:

$$\begin{aligned} l_k(x) &= \vartheta(x_k) + \Delta(x - x_k) + \frac{1}{2}(x - x_k)G(x_k)(x - x_k) \\ &= \frac{1}{2V}(x) + (p(x_k))^T + \frac{1}{2}(x - x_k). \end{aligned} \quad (10)$$

The basic idea of Newton's method is to linearize the system of nonlinear equations, so as to determine the boundary of the system of nonlinear equations with bounded constraints, and obtain

$$x_{k+1} = x_k + \left[ p(x_k)^T p(x_k)^{-1} p(x_k)^T V(x) \right], \quad (11)$$

where  $p(x_k)$  represents the Jacobi matrix of  $V$  at  $x_k$ . When  $c_k = -p(x_k)^{-1} V(x_k)$  represents in the Gaussian Newtonian direction at  $x_k$ , and when the determined boundary value is close to the solution, the Gauss Newton iterative

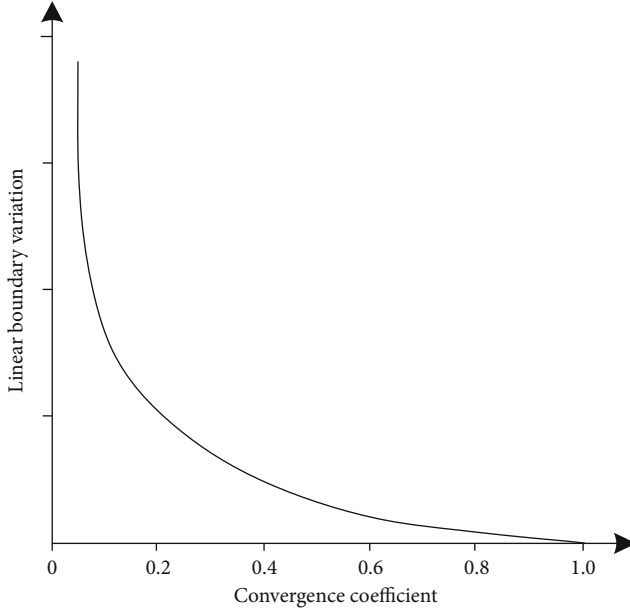


FIGURE 2: Convergence change state of Gauss Newton iterative process.

process can reduce the convergence speed of the nonlinear equation with bounded constraints [16], and its linear convergence state is shown in Figure 2.

According to the above calculation flow and the convergence change state of Gauss Newton iterative process in Figure 2, the bounded variable constrained nonlinear equations are analyzed, and the typical characteristics of a good trust region interior point algorithm are obtained. It has global convergence and fast convergence rate.

**2.2. Trust Region Algorithm.** According to the bounds of the nonlinear equations with bounded constraints determined above, this paper continues to study the trust region algorithm in the nonlinear equations with bounded constraints and determines the region where the points in the trust region of the nonlinear equations with bounded constraints exist. Trust region algorithm is also known as the step control method, originated from Powell's work in. It was first used to solve unconstrained optimization problems and then gradually applied to optimization problems with constraints. It is an important research direction in the field of optimization in recent years [17]. The traditional line search method often fails in solving ill-posed problems because the selected step size is too large. However, the trust region algorithm can effectively solve both good and ill-posed problems, with fast and reliable convergence, and has a good effect on the selection of variable iteration step size.

The basic idea of the trust domain method is to limit the test step  $h_k$  to the trust domain, and the positive number  $\Delta k$  is given at each iteration and requires the test step  $h_k$  to satisfy  $\|h_k\| \leq \Delta k$ . Solve an approximate problem of the original objective function in the trust region and obtain the solution as a trial step. This approximate problem is often called a subproblem [18]. Generally, the trust region subproblem of

unconstrained optimization problem can be

$$\begin{aligned} \min_{d \in R^n} \|V_k + w_k\|^2 = l_k(h), \\ \text{s.t. } \|h\| \leq \Delta k, \end{aligned} \quad (12)$$

where  $V_k = F(x_k)$ ,  $w_k = w(x_k)$ ,  $\Delta k > 0$  is the radius of the current trust domain, and setting  $h_k$  is the solution of the above problem and then obtains

$$\text{preh}_k = l_k(0) - l_k(h_k). \quad (13)$$

The result obtained at this point is the estimated decrease of the objective function, because  $l_k(d) \approx l_k(h_k)$ . The real decrease of the objective function  $F(x)$  is

$$h_k' = \|F_k\|^2 - \|F(x_k + h_k)\|^2. \quad (14)$$

The ratio of real decline to estimated decline is

$$\varphi_k = \frac{\text{preh}_k}{h_k'}. \quad (15)$$

At this point, the larger  $\varphi_k$  represents, the more the target function drops, and the more new iterations there are; so, you can consider expanding the data. When the hour of  $\varphi_k$  is more small, the corresponding points in the trust area are also less [19]. Trust region method has the characteristics of global convergence and fast local convergence.

In Figure 3, in the multidimension, a face  $a(t, c)$  can be regarded as a line, and the normal vector  $t$  is perpendicular to the line and points to the acceptable area of the face. The pyramid is composed of two nonparallel two-dimensional small planes [20]. Figure 3(a) shows such a pyramid. The acceptance area of the pyramid is the intersection of the two acceptance areas of the small planes  $a_1$  and  $a_2$ , which is represented by the intersection area  $Y$  in Figure 3(a). In Figure 3(b), a new small plane intersects the pyramid at two generation points  $q_1$  and  $q_2$ , which are the real entry points [21]. Since the arrow of the normal vector points in the same direction, according to its direction,  $Y$  is also located in the receiving area, and the area covers the area of the pyramid. In Figure 3(c), the normal vector remains unchanged, but the generation point of the two faces is in the position shown in the figure. This is a reduced area, and the receiving area  $Y$  at this time is a smaller area than the original one. The area  $y$  here is no longer the original accepted area, the area is reduced, and the original point is deleted; so, it can be considered to be rejected by  $a$ . In Figure 3(d), the normal vector is pointing in the opposite direction, and the acceptable area is still reduced; so, it can be deleted [22]. The acceptable area intersects the acceptable area of the pyramid; so, the whole acceptable area is reduced to the enclosed area  $Y$ . From the planning problem, it is the constraint conditions to be met jointly [23]. It must be in dry area  $Y$ .

Based on the acceptable domain determined above, select a reasonable acceptable domain for each pyramid

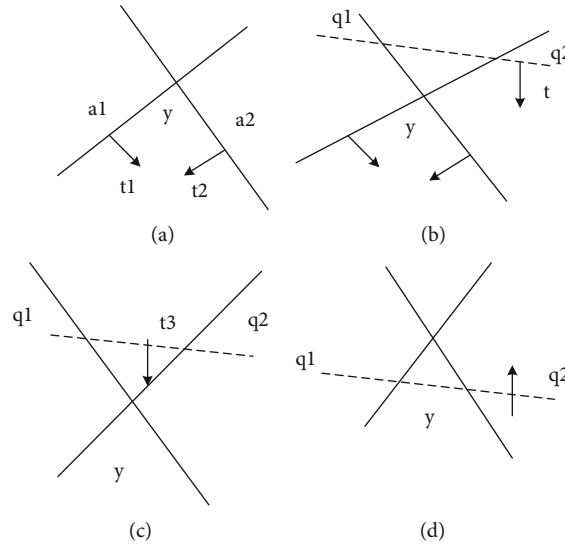


FIGURE 3: Schematic diagram of cone acceptable region.

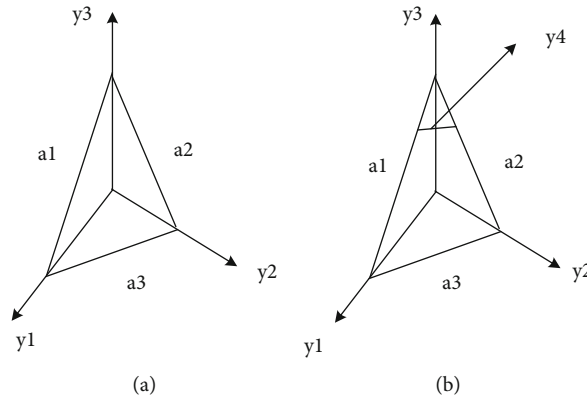


FIGURE 4: Schematic diagram of new acceptable domain.

from  $(m + n)$  and interpret it from two dimensions. However, inequality constraints require three-dimensional transformation under multiple parameters. Therefore, there can be three-dimensional interpretation from two dimensions to three dimensions. When did similar generation methods exist in the previous study of three-dimensional [24]? As shown in the following figure, at this time, they are faceted. Different acceptance domains can be formed for different  $t$  directions. Determine the new acceptable domain, as shown in Figure 4.

In Figure 4(a), the original 3D is the whole area, and the enclosed three-cone structure formed under the constraint of  $a1 : y1 + y3 \leq 1$ ,  $a2 : y2 + y3 \leq 1$ , and  $a3 : y1 + y2 \leq 1$  is the acceptable area, so that the best advantage is to find in such a limited area. The “<1” here is that the direction is vertical  $A$  to  $Y2$ , if “>” will be the opposite direction, and then the area will be different [25], which is similar here so do not repeat. If in Figure 4(b), another constraint appears:  $a$ , the original space is divided into new ones, and then  $a4 = y1 + y2 + 3y3 \leq 1$  new receptive field is gradually formed. Of the temporary, most advantages will become a new vertex of the receptive field; in many inequality constraint nonlin-

ear programming problem, there are a variety of parameters and inequality; so, how to unify the face combination cone, then generated by the cone acceptable domain, this two dimensional matrix table gives enlightenment, if appear “>” is the opposite direction.

### 3. Design of the Trust Region Interior Point Algorithm for Nonlinear Equations with Bounded Variables under Edge Computation

In the design of the trust region interior point algorithm for bounded variable constrained nonlinear equations, the interior point in the acceptable region is determined by the primal dual interior point method. The primal dual interior point method is also known as the path tracking method or tracking center trajectory method. In essence, it is an algorithm that combines logarithmic barrier function, Lagrange function method, and Newton iteration method for optimization calculation by introducing relaxation variable and Lagrange multiplier vector and has the advantages of these three methods at the same time. The primal dual

interior point method starts from the feasible initial interior point and searches for optimization within the feasible region along the steepest descent direction of the primal dual path until it approaches the optimal solution [26]. When solving large-scale linear programming problems, it has been proved theoretically that it has polynomial time complexity and second-order convergence characteristics. It has the advantages of insensitive calculation time to the scale of the problem, stable number of iterations, fast convergence speed, good robustness, and high accuracy.

Firstly, a trust region interior point programming model of bounded variable constrained nonlinear equations is designed; that is,

$$\begin{aligned} \min \chi^T x, \\ s.t. b \leq ax \leq c. \end{aligned} \quad (16)$$

The model is a nonlinear pattern, where  $\chi$  is the order matrix, and  $a, b, c$  represents the number of different state variables and control variables, respectively.

Turning the above model into a simple form, we obtain

$$\begin{aligned} \min \chi^T x, \\ s.t. b \leq ax \leq c, \\ x - 1 = x, \longrightarrow \\ x + u = \bar{x}. \end{aligned} \quad (17)$$

Then, the logarithmic barrier (obstacle) penalty function is introduced into the objective function to transform the objective function into an obstacle function. As shown in the formula, the newly constructed function is close to the original objective function within the feasible region, and its value increases rapidly when it is close to the boundary of the feasible region. Therefore, the minimum value satisfying the condition can only be found in the feasible region, so as to ensure that the optimization is always found within the feasible region in the iterative process [27]; that is,

$$\xi_i = \chi^T x - u \sum_{i=1}^n \ln s_i - u \sum_{i=1}^n \ln s_i - u \sum_{i=1}^n \ln s_i, \quad (18)$$

where  $u \geq 0$  is called a perturbation factor or a barrier (barrier) parameter, and when  $u \rightarrow 0$ , the optimal solution of the formula is equivalent to an optimal solution of the original linear programming problem.

According to the programming model of the trust region interior point of the bounded variable constrained nonlinear equations, the optimal solution can determine the existence state of the interior point in the trust region, but there are still some errors. Therefore, this paper introduces the edge calculation method to correct the error. Edge computing is based on a virtualization platform, and its method is a supplement to nfv; in fact, when NVF focuses on network functions, MEC framework enables applications to run at the edge of the network [28]. The infrastructure carrying MEC and nfv or network functions is very similar; therefore, in

order for operators to get as much benefit from their investment as possible, it will be beneficial to maximize the reuse of nfv infrastructure and infrastructure management by hosting vnfs (virtual network functions) and mec applications on the same platform. The edge computing environment is characterized by low latency, proximity, high bandwidth, real-time insight into radio network information, and location awareness. All this can be translated into value, creating opportunities for mobile operators, applications, and content providers to play complementary and profitable roles in their respective business models, and enable them to better benefit from the mobile broadband experience.

In fact, the concept of edge computing can be traced back to 1998. A content distribution network (CDN) was proposed. It installs cache server in various places and directly connects users' access to the nearest cache server through functional modules such as load balance, content distribution, and scheduling of the central platform, so as to reduce the access pressure and delay of the core network and improve the hit rate. It is a cache network. The difference between CDN and mobile edge computing is that CDN emphasizes the caching of data, while edge computing emphasizes the caching of functions, and gives some functions on mobile devices to MEC. In the early stage, marginalized data were rare, and computers were far from universal [29]. However, with the rapid development of the Internet, in the context of mobile communication and Internet of things, the number of networking devices and mobile devices has ushered in an explosive growth, followed by the explosive growth of fragmented data information generated at the edge of the network. Cloud computing, in this case, cannot fully meet the requirements. Then, satvanarayanan put forward the concept of cloud let. Cloud let is a server that does not need to worry about various resources (such as computing resources, storage resources, and network resources) and is very trusted. It can provide services to users like cloud. However, cloud let emphasizes downlink services, and its main function is to reduce bandwidth and delay. The schematic diagram of edge computing architecture is shown in Figure 5.

According to Figure 5, the constraint conditions to be satisfied need to be set in the trust region interior point error of the boundary calculation correction bounded variable constrained nonlinear equations. One of the first constraints to be met is the limited computing resources at the MEC end. Because the MEC of mobile edge computing cannot have the computing performance of traditional cloud computing, its computing resources are also limited, and it is impossible to handle too many tasks [30]. From this, we can get

$$\sum_{i=1}^n x_{ij} f_i \leq F_i, \forall i \in H. \quad (19)$$

In the formula, the total computational resources of the MEC are expressed as  $f_i$ , namely, the number of points in

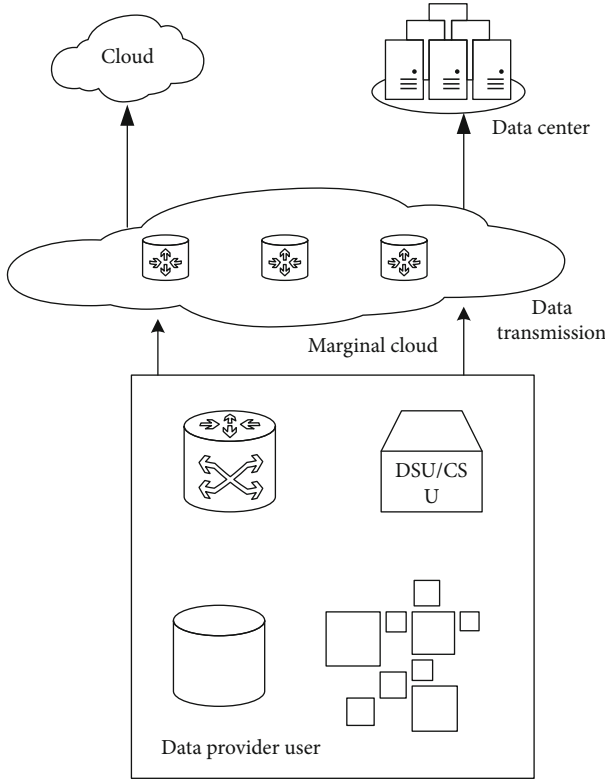


FIGURE 5: Edge calculation framework.

the trust domain. Indicates the total number of tasks that need to be uploaded.

Secondly, assuming that only one task per UE waits to be completed, that is, to determine the existence form of points in the trust domain, it can be obtained:

$$\sum_{i=1}^n x_{ij} f_i \leq 1, \forall_i \in U. \quad (20)$$

Among them,  $x_{ij}$  indicates that the sum of the number of tasks in all points in the trust domain cannot be greater than 1. Since this paper assumes that only one task needs to be uninstalled to the cloud and not local computing, the points in the trust domain are either confirmed or failed; that is,

$$x_{ij} \in [0, 1], \forall_i \in L. \quad (21)$$

In addition to the above constraints, there is also one of the most important constraints, which is the task. The delay of all tasks includes the time of uploading tasks and the time of computing tasks. Since the amount of data returned from computing results is usually very small and the time is very short, regardless of the return time of tasks, the following formula can be obtained:

$$\sum_{i=1}^n x_{ij} (w_{ij}^c + w_{ij}^d) < w_{ij}^{\max}, \forall_i \in L. \quad (22)$$

After calculating the constraint problem according to the set edge, calculate the objective function, that is, the optimi-

zation objective and constraint at the same time in the trust region interior point error of the bounded variable constrained nonlinear equations to obtain the final optimization problem, and obtain the following:

$$\begin{aligned} \sum_{i=1}^n x_{ij} (w_{ij}^c + w_{ij}^d) &\leq w_{ij}^{\max}, \forall_i \in L, \\ x_{ij} &\in [0, 1], \\ P : \text{Minimize} &(E(x) - J(x)). \end{aligned} \quad (23)$$

It can be seen from the above formula that there are two objective functions in the design of the trust in-domain point algorithm of bounded variable constrained nonlinear equations.  $\text{Minimize}(E(x))$  is the best determining speed and convergence in the whole edge calculation process, which can improve the convergence of the trust domain points of bound variable constrained nonlinear equations.

Based on the above edge calculation, the trust region interior point algorithm design of bounded variable constrained nonlinear equations is completed. The specific steps are as follows:

Step 1. Select the initial point  $a_0$ , the trust radius is set to  $\Delta_0 > 0$ , and the maximum trust radius is  $\Delta_{\max} > \Delta_0$ .

Step 2. Initialize the filter subset and set  $K = 0$ .

Step 3. Calculate the inner point trust domain subproblem to obtain the search direction  $O_K$ .

Step 4. Regeneration line search: set  $\alpha_k = 1, l = 0$ , calculate  $x_k(\alpha_k) = x_k + \alpha_k O_k$ , and test whether the iteration point is accepted by the filter; if not, the iteration step is rejected.

Step 5. In order to avoid the use of classical value function, instead of the filter method, make the constraint violation function used that is  $|S(x)|$ . Call point  $d_k$  dominate point  $d^l$ , if and only if  $|S(x)| < \|s(d^l)\|, f(d_k) \leq f(d^l)$ , and definition filter is a list with form  $|S|, f$ , recorded as  $F$ , so that

$$\|s(d^l)\| \leq \|s(d_k)\|, f(d_k) \leq f(d^l). \quad (24)$$

This time holds for all  $k \neq l$ .

Step 6. Update the trust domain radius to get the followings:

$$\begin{aligned} \text{arrd}_k(S(x)) &= f(d_k) - f(d_k + d^l), \\ \text{pred}_k(S(x)) &= \varepsilon_k(0) - \varepsilon_k(D_k, s_k), \\ G &= \frac{\text{arrd}_k(S(x))}{\text{pred}_k(S(x))}. \end{aligned} \quad (25)$$

Step 7. Ensure the existence of points in the trust domain, set  $s_k = \alpha_k d_k$ , and then exists  $x_{k+1} = x_k + s_k$ . Complete the design of the trust domain point algorithm for bounded variable constrained nonlinear equations. The trust region interior point algorithm mainly studies the establishment of trust region subproblem by introducing constraint violation function based on edge calculation. The super-linear convergence rate of the algorithm is guaranteed

without strict complementary relaxation conditions and reasonable assumptions.

In the algorithm design, the interior point in the acceptable region is determined by the primal dual interior point method, the relaxation variable and Lagrange multiplier vector are introduced, the logarithmic barrier function, Lagrange function method, and Newton iteration method are combined to optimize the interior point in the acceptable region, and the trust region interior point programming model of bounded variable constrained nonlinear equations is designed with the help of edge calculation to realize the algorithm design.

#### 4. Example Analysis

*4.1. Design of Experimental Scheme.* For the trust region interior point algorithm of nonlinear equations constrained by bounded variables under the above edge calculation, the mathematical software MATLAB is used to program and realize the specific numerical results. To test the validity of the algorithm, the parameters are selected as follows:  $\varepsilon = 10^{-8}$ ,  $p = 0.4$ ,  $e = 0.02$ , and  $w = 0.05$ .

In the numerical experiment, the following standard numerical tests are selected, and the numerical results are given by the table. NF represents the number of calculations of the objective function, and NG represents the number of its gradient function. The details of the standard numerical tests are shown in Table 1:

According to the standard numerical test questions in Table 1, experimental analysis is carried out on this basis to further analyze the global convergence and local convergence rate of trust region interior points of bounded variable constrained nonlinear equations.

*4.2. Analysis of Experimental Results.* Firstly, the algorithm designed in this paper is used to calculate the actual numerical value of the interior point on the basis of the standard test questions designed in the experimental scheme. The results are shown in Table 2:

The results in Table 2 are in line with expectations. In this example, the iterative sequence can quickly converge to the solution. Although the values in Table 2 are singular, the effect of the algorithm is still very good, because the function considered in the algorithm guarantees the local error bound condition in a field of the solution set, and when the number of iterations of the example increases, the number of times used in the projection gradient direction also increases. This feature shows that the method proposed in this paper is better than the projection gradient method in calculation.

To avoid excessive internal iteration subroutines within 1000 iteration that requires  $NC < 1000$ , the regular parameter update scheme is as follows:  $u_k = \min \{s_k, \|F_k\|^\alpha, s\}$ , where the  $s > 0$ . Parameter is selected as follows:

$$\alpha = 1, s_k = 0.8, \tau = [0, 2.0], \beta = 0.8, p = 0.7, s = 0.001. \quad (26)$$

Based on the above parameter setting, the numerical results of the global convergence of the interior points in

TABLE 1: Standard numerical test questions.

Number	Content
1	(HS003): $f = x_2 + 0.00001(x_2 - x_1)^2$
2	(HS004): $f = 1/3(x_1 + 1)(x_2 + 1)(x_2 - x_1)^2$
3	(HS005): $f = \sin(x_1 + 1)(x_2 + 1)(x_2 - x_1)^2$
4	(HS006): $f = 4(x_1 - 1)(x_2 - 1)(x_2)^2$
5	(HS007): $f = 1000(x_1 + 1)(x_2 - x_1)^2$

TABLE 2: Numerical results of this algorithm.

Title number	NF	NG
1	3	3
2	3	3
3	6	7
4	2	2
5	5	5

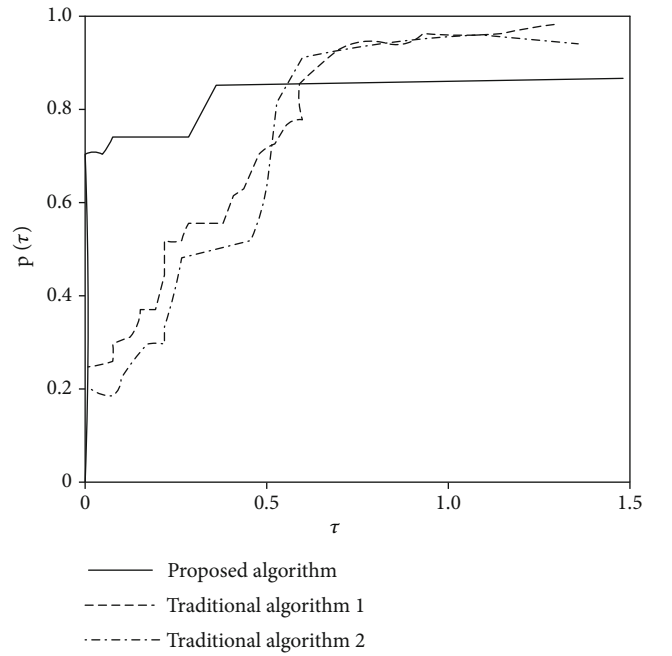


FIGURE 6: Comparison of global convergence changes of trust region interior points of different algorithms.

the trust region of the bounded variable constrained nonlinear equations are analyzed. By comparing the traditional algorithm 1 and the traditional algorithm 2, and characterized by the performance graph, the global convergence results are shown in Figure 6.

By analyzing the experimental results in Figure 6, it can be seen that when the  $\tau$  value changes, the overall convergence of the trust region of the proposed algorithm and the two traditional methods shows different fluctuations. Among them, when the value is about 0.5, the overall convergence of the internal points in the trust region of the proposed algorithm changes smoothly; that is, the convergence



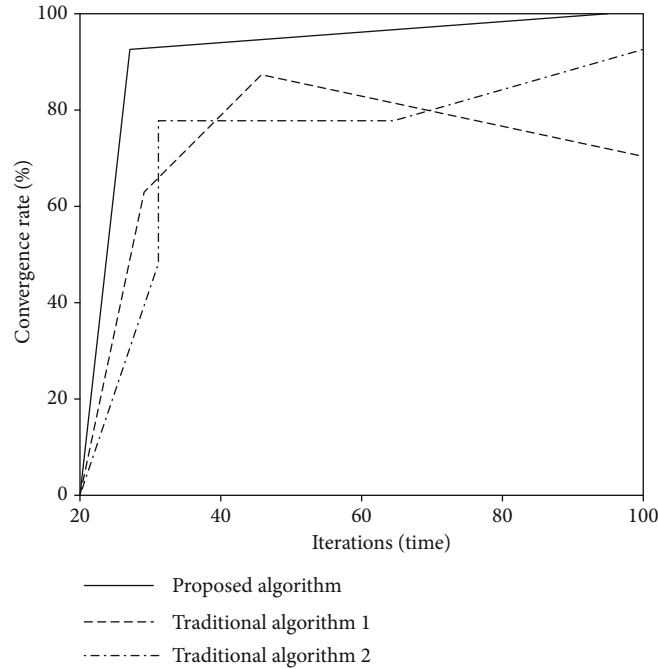


FIGURE 7: Variation of local convergence rate of points in trust region of nonlinear equations with bounded variable constraints.

is relatively stable, while the changes of the other two traditional algorithms also show a fluctuating trend. It can be seen that the overall convergence of the internal points in the trust region can be effectively improved by using this algorithm.

Based on the above experiments, the local convergence rate of the interior point of the trust region of the system of nonlinear equations constrained by bounded variables is analyzed. The numerical results are shown in Figure 7.

By analyzing Figure 7, it can be seen that with the continuous updating of the number of iterations, there are certain differences in the local convergence rate of the trust region of the three methods for the bounded variable constrained nonlinear equations in the example. Among them, the best local convergence rate in the trust region of nonlinear equations with bounded variables is the method in this paper, which can be 100% at the highest, while the local convergence rates of the other two methods show an unstable state, which verifies the effectiveness of the proposed algorithm.

To sum up, compared with other algorithms, the overall convergence of the trust region interior point algorithm of the bounded variable constrained nonlinear equations under the designed edge calculation fluctuates relatively smoothly. There are some differences in the local convergence rate of the trust region interior point of the bounded variable constrained nonlinear equations in the example, and the convergence rate of this method is the best.

## 5. Conclusion

The trust region interior point algorithm for nonlinear equations with bounded variable constraints is a key algo-

rithm in the field of mathematics. It has the problems of low global convergence and low local convergence rate in the existing research methods. Therefore, the trust region interior point algorithm for nonlinear equations with bounded variable constraints under edge calculation is designed. By constructing the basic function form of nonlinear equations constrained by bounded variables, the Gauss Newton iterative process is used to determine the boundary of nonlinear equations and ensure the global convergence of their changes. The trial step is limited to the trust region, the original objective function is solved, the trust region subproblem of the unconstrained optimization problem is analyzed, and an acceptable region is generated by a cone. The interior point in the acceptable region is determined by the primal dual interior point method, the relaxation variable and Lagrange multiplier vector are introduced to optimize the interior point in the acceptable region, and the trust region interior point programming model of bounded variable constrained nonlinear equations is designed with the help of edge calculation. The research results show that the quality of the trust region interior point algorithm of bounded variable constrained nonlinear equations can be improved by using the designed algorithm. There is an objective function in the design of the trust region interior point algorithm for bounded variable constrained nonlinear equations, which has the best speed and convergence in the whole edge calculation process, and can effectively improve the convergence of trust region interior points of bounded variable constrained nonlinear equations. However, there are many constraints and changing conditions in the research of mathematical methods. Therefore, the algorithm research under more constraints will be studied in the future.

## Data Availability

The author can provide all the original data involved in the research.

## Conflicts of Interest

The authors indicate that there was no conflict of interest in the study.

## References

- [1] J. Dahl and E. D. Andersen, "A primal-dual interior-point algorithm for nonsymmetric exponential-cone optimization," *Mathematical Programming*, vol. 17, no. 2, pp. 332–337, 2021.
- [2] X. E. Chouzenou, M. C. Corbineau, and J. C. Pesquet, "A proximal interior point algorithm with applications to image processing," *Journal of Mathematical Imaging and Vision*, vol. 62, no. 6-7, pp. 919–940, 2020.
- [3] L. Scheunemann, P. S. Nigro, J. Schröder, and P. M. Pimenta, "A novel algorithm for rate independent small strain crystal plasticity based on the infeasible primal-dual interior point method," *International Journal of Plasticity*, vol. 124, no. 5, pp. 1–19, 2020.
- [4] K. Yoshimoto, "A primal-dual interior-point method for facility layout problem with relative-positioning constraints," *Algorithms*, vol. 14, no. 12, pp. 96–105, 2021.
- [5] J. Castro and S. Nasini, "A specialized interior-point algorithm for huge minimum convex cost flows in bipartite networks - ScienceDirect," *European Journal of Operational Research*, vol. 6, no. 12, pp. 74–80, 2020.
- [6] H. H. Dwaib and M. Shiker, "Using trust region method with BFGS technique for solving nonlinear systems of equations," *Journal of Physics: Conference Series*, vol. 1818, no. 1, pp. 12–22, 2021.
- [7] E. Nagy and A. Varga, "A new Ai-Zhang type interior point algorithm for sufficient linear complementarity problems," *Corvinus Economics Working Papers (CEWP)*, vol. 14, no. 2, pp. 463–473, 2022.
- [8] W. Grimes, "Path-following interior-point algorithm for monotone linear complementarity problems," *Asian-European Journal of Mathematics*, vol. 26, no. 2, pp. 72–77, 2021.
- [9] M. Yamashita, E. Iida, and Y. Yang, "An infeasible interior-point arc-search algorithm for nonlinear constrained optimization," *Numerical Algorithms*, vol. 36, no. 2, pp. 1–27, 2021.
- [10] J. Huang, J. Fang, L. Hu, B. Shi, S. Li, and Y. Tu, "Reactive power optimization system of distribution network based on edge calculation[J]," *Journal of Physics Conference Series*, vol. 1914, no. 1, pp. 12–33, 2021.
- [11] F. Rahpeymaei, "An efficient line search trust-region for systems of nonlinear equations," *Mathematical Sciences*, vol. 14, no. 3, pp. 257–268, 2020.
- [12] M. S. Shahraki and A. Delavarkhalafi, "An arc-search predictor-corrector infeasible-interior-point algorithm for P( $\kappa$ )-SCLCPs," *Numerical Algorithms*, vol. 74, no. 21, pp. 784–788, 2020.
- [13] M. E. Nagy and R. A. Va, "A new long-step interior point algorithm for linear programming based on the algebraic equivalent transformation," *Corvinus Economics Working Papers (CEWP)*, vol. 145, no. 8, pp. 9634–9637, 2021.
- [14] B. Kheirfam, "A polynomial-iteration infeasible interior-point algorithm with arc-search for semidefinite optimization," *Journal of Scientific Computing*, vol. 88, no. 3, pp. 1–23, 2021.
- [15] B. Kheirfam, "A new predictor-corrector infeasible interior-point algorithm for linear optimization in a wide neighborhood," *Fundamenta Informaticae*, vol. 177, no. 2, pp. 141–156, 2020.
- [16] Z. Darvay, T. Illés, and P. R. Rigó, "Predictor-corrector interior-point algorithm for sufficient linear complementarity problems based on a new type of algebraic equivalent transformation technique," *Corvinus Economics Working Papers (CEWP)*, vol. 4, no. 3, pp. 71–79, 2020.
- [17] D. Ek and A. Forsgren, "Approximate solution of system of equations arising in interior-point methods for bound-constrained optimization," *Computational Optimization and Applications*, vol. 79, no. 1, pp. 155–191, 2021.
- [18] D. Ek and A. Forsgren, "Approximate solution of system of equations arising in interior-point methods for bound-constrained optimization," *Computational Optimization and Applications*, vol. 34, no. 21, pp. 741–749, 2020.
- [19] Z. Sabir, M. Raja, A. Kamal et al., "Neuro-swarm heuristic using interior-point algorithm to solve a third kind of multi-singular nonlinear system," *Mathematical Biosciences and Engineering*, vol. 18, no. 5, pp. 5285–5308, 2021.
- [20] K. U. Danmalam, H. Mohammad, A. B. Abubakar, and A. M. Auwal, "Hybrid algorithm for system of nonlinear monotone equations based on the convex combination of fletcher-reeves and a new conjugate residual parameters," *Thai Journal of Mathematics*, vol. 18, no. 4, pp. 2093–2106, 2020.
- [21] C. S. Wang, Y. F. Yan, Q. Xu et al., "Edge sensor based real-time calculation method for active panel position of QTT[J]," *Research in Astronomy and Astrophysics*, vol. 20, no. 11, pp. 273–282, 2020.
- [22] V. A. Ramirez and G. N. Sottosanto, "Nonmonotone trust region algorithm for solving the unconstrained multiobjective optimization problems," *Computational Optimization and Applications*, vol. 47, no. 3, pp. 987–992, 2022.
- [23] E. Hung and M. J. Kao, "Approximation algorithm for vertex cover with multiple covering constraints," *Algorithmica*, vol. 84, no. 1, pp. 1–12, 2021.
- [24] K. Yamaguchi and J. Templin, "A Gibbs sampling algorithm with monotonicity constraints for diagnostic classification models," *Journal of Classification*, vol. 11, no. 1, pp. 854–859, 2022.
- [25] S. A. Karbasy, A. Hamdi, M. Salahi, and A. Taati, "An efficient algorithm for large-scale extended trust-region subproblems with non-intersecting linear constraints," *Optimization Letters*, vol. 15, no. 4, pp. 1425–1446, 2021.
- [26] L. Chao, J. Han, L. Yan et al., "Fast compressed sensing analysis for imaging reconstruction with primal dual interior point algorithm," *Optics and Lasers in Engineering*, vol. 129, no. 6, pp. 106082–106122, 2020.
- [27] M. E. Nagy and A. Varga, "A new Ai-Zhang type interior point algorithm for sufficient linear complementarity problems," *Corvinus Economics Working Papers (CEWP)*, vol. 3, no. 3, pp. 1–10, 2022.
- [28] D. Liu and Q. Tran-Dinh, "An inexact interior-point Lagrangian decomposition algorithm with inexact oracles," *Journal of Optimization Theory and Applications*, vol. 185, no. 3, pp. 903–926, 2020.

- [29] O. K. Oyewole, H. A. Abass, and O. T. Mewomo, "Strong convergence algorithm for a fixed point constraint split null point problem," *Rendiconti del Circolo Matematico di Palermo*, vol. 35, no. 2, pp. 6378–6381, 2020.
- [30] Y. Zhang and H. Liu, "A Barzilai and Borwein regularization feasible direction algorithm for convex nonlinear SOC programming with linear constraints," *Journal of Computational and Applied Mathematics*, vol. 14, no. 21, pp. 637–642, 2021.